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SUPPLY CHAIN MANAGEMENT FOR MULTI-ECHELON INVENTORY MODEL

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ABSTRACT:

We consider a multi-echelon model for analyzing the performance of a supply-chain with a single manufacturer and multiple retailers. A single manufacturer supplies I identical products to N identical retailers. The demand for individual product for different retailers is independent, identical, and general distributed. The manufacturer waits for q orders to be accumulated and then produces multiple products in batches of q units. The effects of batch size on manufacture's lead time, and the effect of product variety on inventory cost are studied. The multi-echelon model allows us to examine the effect of processing time, setup time, product variety, demand rate, and the number of retailers on the inventory cost directly.

Keywords: Multi-echelon, Supply-chain, Batch order, Inventory model, Setup time.

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INTRODUCTION

Management decisions arise out of the productive nature of organizations. In a general sense, it could be suggested that the improvement of productivity is the basic motivation behind all management decision making. Productivity, therefore, can be considered as a very general concept of the relation between the outputs and the inputs of a productive system.

Supply chain management (SCM) covers planning and management of all activities involving in sourcing and procurement, conversion, and all logistic management activities. It also includes coordination and collaboration with channel partners, which can be suppliers, intermediaries, third-party service providers, and customers. In essence, supply chain management integrates supply and demand management within and across the companies. SCM is an integrated function with primary responsibility for linking major business function and business processes within and across companies into a cohesive and high-performing business model. It includes all of the logistics management activities, as well as manufacturing operations, and it drives coordination of processes and activities with and across marketing, sales, product design, and finance and information technology.

Most of the literature on inventory system assumed that the items are consumables. Nevertheless, the significance of recoverable items in terms of the amount of money invested in stock has led to recent research efforts to model inventory systems for these items. Examples of past studies include the works of Sherbrooke [24], and Moinzadeh and Lee [19]. Van Beek [29] gave a heuristic analysis of the continuous review, fixed order sized (b, Q) model where a fixed amount Q is ordered each time the inventory position drops below the level b. His analysis was based on the replenishment cycle approach.

It is generally assumed that an arriving batch of defective items is either repairable or non-repairable in a batch, i.e., the cases where a portion of the batch is repairable and the rest items are non-recoverable. Considerable work has also been done for multi-echelon inventory systems where the item can be recoverable with a certain probability. For example, Simon [26] considered a case with Poisson demands and deterministic lead times for repair and procurement, which was later, extended to the compound Poisson case by Shanker [23].

Basic analysis of the (R, S) inventory system can be found in many standard textbooks (cf. Hadely and Whitin [10] and Silver and Peterson [25]). These studies mainly focus on the computation of certain performance measures and the choice of optimal values for R. To capture customers in different market segments, many firms distribute their products using multiple channels involving in-store, mail order and electronic mediums. Nevertheless, the business literature is replete with examples where the management of multiple distribution channels becomes unwieldy. As a way to evaluate different distribution strategies arising from multiple channels, Tang and Alptekinoglu [27] developed a model of a general multi-channel distribution system subject to stochastic demand.

Bhattacharya [4] gave a new approach towards a two–item inventory model for deteriorating items with a linear stock–dependent demand rate. In fact, for the first time, the interacting terms showing the mutual increase in the demand of one commodity due to the presence of the other, was accommodated in the model. Lin and Lin [16] proposed a two-echelon inventory model for a periodical commodity, in which the market and manufacturing channels are combined. This model can be used to solve the production policy, the order policies of the raw materials for the manufacturer, and order size for the retailer. Mc Tavish and Goyal [18] considered an integrated marketing and production decision planning. Rao [21] suggested the joint determination of optimal inventory and marketing policy. Rosenblatt et al. [22] considered the problem of developing an acquisition policy; specifically, given a set of potential suppliers, from whom should the firm buy the product, in what quantities, and how often?

Even under the best of circumstances, retail inventory planning is a complicated task. The task is particularly formidable since the firm must often contend with many hundred or even thousands of unique stock items where the individual sales levels are characterized as slow-moving or low usage [1]. Numerous examples of this type of inventory planning environment exist in both the public and private sectors including clothing stores, gift shops, bookstores, record outlets, hospitals and spare parts inventory depots [6]. The traditional inventory control decisions concerning the timing and sizing of order are still relevant whereas the traditional methodologies of the order point and order quantity models are not [8]. Archibald and Silver [3] analyzed the (s, S) policies under continuous review and discrete compound Poisson demand. Chatfield and Goodhardt [5] studied a consumer-purchasing model with Erlang inter-purchase times. Graves [9] studied a multi-echelon inventory model for a repairable item with one-for-one replenishment. Thonemann and Bradley [28] presented a stylized model for analyzing the effect of product variety on supply-chain performance for a supply chain with a single manufacturer and multiple retailers. Importance of supply chain coordination by a systemetic literature review is presented by Kanda and Deshmukh [15]. Three different types of channel structure (decentralized, semi-integrated and integrated) is studied by Huang and Huang [14]. Uncertainty of demand in multi echelon supply chain model is studied by many authors. (He and Zhao [11], Hu et al. [13], Wang [30], etc..). Again for stochastic demand, a single supplier single manufacturer supply chain management problems of production and procurement is considered by Xu [31]. Supply chain management from vendor side for multi retailer is studied by Mateen et al. [17] under stochastic demand. Almeder et al. [2] considered lead time in multi-level business problem. A decentralized supply chain consist of one manufacturer (manufacturer has random yield) and one retailer (retailer faces uncertain demand) is studied by Yin and Ma [32]. Zhao et al. [33] formulated supply chain inventory model as a mixed integer nonlinear programming problem. Dai et al. [7] considered Multi-echelon inventory in supply chain with reverse ramp-type demand. Hosseinifard and Abbasi [12] analyzed impacts of centralization in a two-echelon supply chain with perishability. Qu et al. [20] studied the incentive problems associated with inventory investment in a supply chain.

The rest of the paper is organized as follows. In section 2, we describe two-echelon supply-chain model by defining requisite assumptions and notations used. The expected lead time is derived in section 3. In section 4, we find the optimal batch size. In section 5, some special cases are deduced. In section 6, cost minimization problem is considered. In section 7, numerical illustration is provided. To examine the effect of different parameters, numerical results are exhibited graphically. Finally in section 8, we summarize our findings, and explain how our model can be further modified, also some promising areas for future research is discussed.

1. Model Discription

We develop a two-echelon supply chain model, in which there is only a single manufacturer and N identical retailers. The notations used to describe our model are given below.

I Number of products

N Numbers of retailers

Λ Aggregate demand rate over all retailers
λ_{in} Demand rate for product *i* at retailer *n*.
T Changeover time
t Unit manufacturer time
q Batch size
u Transportation time from manufacturer to retailers
LT Manufacturing lead time
τ Total lead time to retailer, E[LT]+u
ρ Manufacturing utilization including production and changeover.
C_A^2 Square coefficient of variation of demand.
C_S^2 Square coefficient of variance of supply.
W_B Waiting time in the batch buffer.
W_p Waiting time in the process buffer.
S Service time of a batch.

3. Expected lead time

Suppose that the manufacturer produces I products for N identical retailers. All the I products have same cost, and the demand for each i^{th} (I=1,2,….,I) product by n^{th} (n=1,2,….,N) retailers is independent, identical, and general distributed. The aggregate supply-chain demand rate is given by

$$\Lambda = \sum_{i=1}^{I}\sum_{n=1}^{N}\lambda_{in} = IN\lambda$$

The manufacturer waits for q orders to be accumulated and then produces multiple products in batches of q units. First come first served rule is applied to process the batches of q units. The manufacturing time for each batch of q units is T+qt, where T is the setup time and t is unit manufacturer time, both manufacturing and set up times are deterministic and do not depend on the product produced. Let the time consumed in transportation from manufacturer to retailer be u.

The expected lead time, of an order is, given by

$$E\,[LT] = E\,[W_B+W_p+S] \qquad \dots(1)$$

It is assumed that the manufacturer, which is not always optimal, uses constant batch sizes. It is taken into consideration due to mathematical tractability.

The expected waiting time in the batch buffer is

$$E\,[W_B] = \frac{(q-1)I}{2\Lambda} \qquad \dots(2)$$

The average service time of a batch is given by

$$E[S] = T + qt \quad \ldots(3)$$

Here we consider a queueing system, which is equivalent to the manufacturing process and distribution. For this purpose G/G/1 model is used to derive the average time an order spends in the process buffer. Now the average time an order spends in the process buffer is given by

$$E[W_p] = \frac{\Lambda(T+qt)^2}{2[q(1-\Lambda t)-\Lambda t]}\left(C_A^2 + C_S^2\right) \quad \ldots(4)$$

Equation (4) gives a closed-form expression for the expected time an order spends in the system. The expected time an order spends in the queueing system and the transportation time from the manufacturer to the retailer, make together the expected replenishment lead time of our supply-chain model.

So, the expected lead time is obtained by using (1) – (4) as

$$E[LT] = \frac{q-1}{2\Lambda} I + \frac{\Lambda(T+qt)^2}{2[q(1-\Lambda t)-\Lambda T]} A + T + qt \quad \ldots(5)$$

where A= $\left(C_A^2 + C_S^2\right)$

The minimal expected lead time is

$$\tau = \min \frac{q-1}{2\Lambda} I + \frac{\Lambda(T+qt)^2}{2[q(1-\Lambda t)-\Lambda T]} A + T + qt + u \quad \ldots(6)$$

The utilization rate is given by

$$\rho = \Lambda\left(\frac{T}{q} + t\right) \quad \ldots(7)$$

4. Optimal policy

The objective of our investigation in this section is to minimize the expected lead time so as to determine the optimal batch size q. For this purpose, we use classical approach as follows:

Consider $\frac{\partial \tau}{\partial q} = 0$ which gives

$$q_1^* = \frac{\Lambda T}{(1-\Lambda t)}\left[1+\sqrt{\frac{A}{R}}\right], \quad \ldots(8)$$

and

$$q_2^* = \frac{\Lambda T}{(1-\Lambda t)}\left[1-\sqrt{\frac{A}{R}}\right] \quad \ldots(9)$$

where, $R = A\Lambda^2 t^2 + 2\Lambda t(1-\Lambda t) + I(1-\Lambda t)$ …(10)

It is evident that q_2^* is not feasible because it suggests a utilization rate greater than one. So, we use q_1^* to determine the optimal utilization rate ρ^* and lead time τ^*.

Thus, the optimal utilization rate is given by

$$\rho^* = \frac{\sqrt{R} + \Lambda t\sqrt{A}}{\sqrt{R} + \sqrt{A}} \quad \ldots(11)$$

Also, the optimal lead time is

$$\tau = \frac{\left[\Lambda T(1-\Lambda t) - (1-\Lambda t)^2\right] + 2\Lambda T\left[A\Lambda t - \Lambda t + \sqrt{AR} + 1\right]}{2\Lambda(1-\Lambda t)^2} + u \quad \ldots(12)$$

5. some special cases

Case 1: If we take c_A^2 =1 and c_s^2 =1, then process buffer is governed by M/M/1 model. In this case, optimal values of different system parameters reduces to:

$$q_1^* = \frac{\Lambda T}{(1-\Lambda t)}\left[1+\sqrt{\frac{2}{R}}\right] \quad \ldots(13)$$

$$\rho^* = \frac{\sqrt{R} + \Lambda t\sqrt{2}}{\sqrt{R} + \sqrt{2}} \quad \ldots(14)$$

$$\tau^* = \frac{\left[\Lambda T(1-\Lambda t)-(1-\Lambda t)^2\right]+2\Lambda T\left[\Lambda t+\sqrt{2R}+1\right]}{2\Lambda(1-\Lambda t)^2}+u \qquad \ldots(15)$$

Case 2: For M/D/1 model for process buffer, we set c_A^2=1 and c_s^2=0. Also for $E_2/E_2/1$ model we have c_A^2=0.5 and c_s^2=0.5. For both models, the optimal batch size, utilization rate, and the optimal expected lead time are given by

$$q_1^* = \frac{\Lambda T}{(1-\Lambda t)}\left[1+\frac{1}{\sqrt{R}}\right] \qquad \ldots(16)$$

$$\rho^* = \frac{\sqrt{R}+\Lambda t}{\sqrt{R}+1} \qquad \ldots(17)$$

and
$$\tau^* = \frac{\left[\Lambda T(1-\Lambda t)-(1-\Lambda t)^2\right]+2\Lambda T\left[\sqrt{R}+1\right]}{2\Lambda(1-\Lambda t)^2}+u \qquad \ldots(18)$$

In this case our results coincide with that obtained by Thenemann and Bradely [20].

Case 3: Setting c_A^2=0.5 and c_s^2=1, we propose E_2/M/1 model for buffer process. In this case the optimal batch size q^*, utilization rate ρ^* and the optimal expected lead time τ^* are given below:

$$q_1^* = \frac{\Lambda T}{(1-\Lambda t)}\left[1+\frac{\sqrt{1.5}}{\sqrt{R}}\right] \qquad \ldots(19)$$

$$\rho^* = \frac{\sqrt{R}+\Lambda t\sqrt{1.5}}{\sqrt{R}+\sqrt{1.5}} \qquad \ldots(20)$$

and
$$\tau^* = \frac{\left[\Lambda T(1-\Lambda t)-(1-\Lambda t)^2\right]+2\Lambda T\left[0.5\Lambda t+\sqrt{1.5R}+1\right]}{2\Lambda(1-\Lambda t)^2}+u \qquad \ldots(21)$$

6. Cost minimization analysis

In this section we discuss the cost minimization problem. For this purpose, we obtain retailor's cost for a given replenishment lead time. Now we define some notations as given below:

C_{in} Unit cost of i^{th} (i=1,2,3,….,I) product at n^{th} (n=1,2,3,…,N) retailer.
C_h Holding cost.

C_p Backorder penalty cost.
z Optimal order up to level
TC Total cost.
Y Limiting distribution of the number of order in resupply

The retailer's cost minimization problem can be stated as

Min TC= $\{C_{in}\lambda + C_h E[s-y]^+ + C_p E[y-s]^+\}$ I …(22)

where

$$[x]^+ = \begin{cases} x & ,x > 0 \\ 0 & ,x < 0 \end{cases}$$

The optimal solution to cost minimization problem in case when replenishment times are i.i.d. with mean τ, is given by

$$z^* = F_{\lambda\tau}^{-1}\left(\frac{C_p}{C_h + C_p}\right) \quad \text{…(23)}$$

where $F_{\lambda\tau}$ is used for c.d.f. of a Poisson distribution with mean $\lambda\tau$. Also

$$z^* = \frac{\tau\Lambda}{IN} + v\sqrt{\frac{\tau\Lambda C_a^2}{IN}} \quad \text{…(24)}$$

where demand is Normally distributed with mean $\lambda\tau$ and the variance $\lambda\tau c_a^2$

and $$v = \Phi^{-1}\left(\frac{C_p}{C_h + C_p}\right) \quad \text{…(25)}$$

Here Φ denotes the cumulative distribution function of standard normal distribution.

Let G(y) be the c.d.f. of a Normal distribution with mean $\lambda\tau$ and variance $\tau\lambda C_a^2$, then

$$E\{C_h(z^* - y) + C_p(y - z^*)\} = C_h \int_{y=0}^{S^*} (z^* - y)dG(y) + C_p \int_{y=S^*}^{\infty} (y - z^*)dG(y)$$

$$= [C_h Z + (C_p + C_h)L(v)]\sqrt{\frac{\tau\Lambda C_a^2}{IN}} \quad \text{…(26)}$$

where

$$L(v) = \int_{\xi=v}^{\infty} (\xi - TC)d\Phi(\xi) \quad \text{…(27)}$$

Using equation (26) in (22), we get the total cost (TC) a given by

$$TC = \frac{C_{in}\Lambda}{N} + \left[C_h Z + (C_p + C_h)L(v)\right]\sqrt{\frac{\tau\Lambda C_a^2}{IN}} \quad \ldots(28)$$

We now analyze the effect of product variety on a retailer's cost when product variety affects lead time. We assume that the number of product I and the number of retailers N in the supply chain are large. These assumptions are necessary to ensure that the expression for expected lead time is accurate and that replenishment lead time is independent. The minimum cost at the end of retailer is obtained by using (21) and (28) as

$$TC^* = r(v)\sqrt{\left[\frac{\left(\Lambda T(1-\Lambda t)-(1-\Lambda t)^2\right)I + 2\Lambda T\left(\Lambda t A + \sqrt{AR} - \Lambda t + 1\right)}{2N(1-\Lambda t)^2} + \frac{u}{N}\right] I C_a^2} + \frac{C\Lambda}{N} \quad \ldots(29)$$

where $r(v) = C_h v + (C_p + C_h)L(v)$

7. numerical results

In this section, numerical results for expected lead time and total optimal cost are calculated using MATLAB software. The graphical presentation has also been done in figures 1-5. In figure 1, we depict the effect of aggregate demand on total optimal cost. It is observed that aggregate demand is concavely increasing with total optimal cost so that as retailers increase their demand rate, profit will be automatically maximized. The effect of number of retailer's over total optimal cost for different level of product variety is shown in figure 2, and it is noted that optimal cost decreases with the increase in the number of retailers. From figure 3, we notice that as changeover time increases the total optimal cost for different level of product variety (I) also increases.

In figure 4, the effect of batch size on the expected lead time for different levels of product variety I is depicted. We observe that with the increase in the batch size, the expected lead time also increases for different levels of product variety (I).

Expected lead time is linearly increasing function of product variety (I) as is clear from figure 5.

8. conclusion

We have explored the effect of different parameters in multi-echelon supply-chain model dealing with single manufacturer and multiple retailers. The manufacturer supplies I identical products to N identical retailers; the manufacturer waits for q orders to be accumulated and then produces multiple products in batches of q units. We have observed the effect of batch size and product variety on expected lead time and find that as we increase the batch size and product variety, the expected lead time will also increase. We also notice that the total optimal cost is concave increasing in aggregate demand and changeover time T.

The investigation done can be extended by considering G/G/1 model for which approximate result for waiting time is known; the work in this direction is in progress.

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